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Near-field tangential particle velocities: piston radiator *vs.* QRD phased linear array

Toby Gifford[1], Malcolm Duffield[2], and Joe Hayes[3]

[1]*Monash University*
[2]*Vastigo Ltd*
[3]*A3D Holdings Ltd*

Correspondence should be addressed to Toby Gifford (`toby.gifford@monash.edu`)

ABSTRACT

Loudspeakers in cars are constrained in size and placement. As such, near-field and mid-field sound properties are of interest. The near-field of a circular piston in an infinite baffle is well studied, and often used as an approximate model for a loudspeaker. Whilst a piston's SPL directivity pattern is well understood, its polar pattern of particle velocity direction has received less attention. We calculate the radial and tangential components of the piston model's particle velocity field, and find that tangential velocity dominates at particular angles for a given piston frequency. These polar positions relate to the 'dips' in the SPL directivity pattern. Contrasting these results with the sound-field of a linear array of spherical sound sources having phase delays determined by a QRD sequence, we find the QRD array yields greater uniformity in both SPL directivity and particle velocity direction.

1 Introduction

It is common to see numerous speaker-drivers in a modern car sound system. The interaction of this plurality of drivers is governed by signal processing methods that seek to provide a controlled acoustic sound field within the confined space.

Given the frequency and polar dependent performance of an individual circular piston loudspeaker (see for example Figure 7.2 in [1]), the volume of air being moved to achieve the desired loudness, and the limitation of useful frequency range of the individual drivers, achieving an even sound field in the confined space of a car interior can be challenging.

This article discusses the potential for a new audio reproduction technology called A3D to mitigate unwanted spatial variation of the sound-field, typically caused by both near-field diffraction and specular reflection. We further suggest that the direction of particle velocity may be perceptually important. In this paper we compare numerical simulations of a cylindrical piston radiator (a simple model of a typical speaker) with a Quadratic Residue Diffuser (QRD) line-array modeled to mimic the functioning of an A3D speaker.

The research presented here arose from studying the sound-field properties of a multi-source array using phase delays derived from a quadratic residue sequence. This proprietary technique 'A3D' [2],[3] was originally developed to mimic musical instrument acoustics and mitigate room reflections in sound reproduction, and was informed by time-domain descriptions of physical sound-diffusion surfaces [4], as well as the pioneering work of Schroeder [5]. This research is part of ongoing efforts to understand the sound-field properties of A3D, specifically in comparison with conventional loudspeakers.

The modeling of both circular piston radiators and line arrays of spherical sources are well established, although particle velocity direction has not typically been the focus of attention. We investigate the particle velocity vectors of the circular piston radiator and compare these to a QRD line-array approach which we call 'diffusion at source'. The QRD method appears to mitigate tangential particle velocities which, we suggest, may benefit audio reproduction within confined specular-reflective spaces.

We compare the modeled sound-field directivity of the QRD-array with experimental measurements of an A3D speaker conducted by Callahan Innovation, and find good agreement in the polar patterns. For the piston model the simulated directivity patterns conform to well-understood and experimentally verified lobe patterns [6].

The particle velocity comparison between circular piston and QRD-array sound fields presented here is based on computational models rather than experimental measurements. At present the authors do not have access to an appropriate particle velocity probe (for example a Microflown) and suggest experimental verification of these particle velocity patterns as an obvious next step in this research.

1.1 Hypotheses regarding perception of space

One of the perceptual effects encountered using prototype A3D 'diffusion at source' systems, is a quite different sense of spatiality to that experienced with conventional loudspeakers. Listeners have commented, in A/B listening experiments, that A3D systems seem to create sounds that 'hang in space'. This research was, in part, pursued in order to understand the physical processes contributing to this perception. Noting that piston-like drivers have significant tangential energy components, we put forward the following hypothesis: for optimal spatial reproduction, tangential acoustic energy should be created only by stereo (or multi-channel) differences.

QRD is a *lateral* diffusion mechanism [5]. An ideally diffuse sound-field has zero net energy transport at all points and in all directions [4]. A *laterally* diffuse sound-field has zero net *tangential* energy transport, or otherwise put has zero tangential sound-intensity. A sound-field in which all particle velocities are radial will be laterally diffuse.

Given that binaural dissimilarity strongly influences spatial perception [7], it seems reasonable to hypothesise that an ideal sound reproduction approach should exhibit a vanishing quiescent lateral sound intensity field. We further suggest that Interaural Time Difference (ITD) should benefit from linear sound paths to both ears and Interaural Intensity Difference (IID) should benefit from a quiescent lateral intensity.

2 Background

Loudspeaker designers are particularly concerned with the radiated sound-field from a circular membrane as an approximate model for the output of a loudspeaker cone. A number of design constraints must be balanced with the objective of faithfully reproducing the input signal with minimal waveform distortion or frequency coloration [8]. Frequency dependent directivity and acoustic impedance matching considerations typically dictate the use of multiple drivers of different sizes (i.e. tweeter, woofer, sub, etc.) [8]. The resultant sound field typically displays complex spatial interference patterns.

These patterns are due to both interference between the speaker-drivers around the crossover frequencies, and also the non-zero tangential-energy area of the individual drivers. Those due to individual drivers of given area have been studied extensively in acoustics, using a range of models [9]. There are known differences between actual radiation patterns of a loudspeaker in cabinet *vs.* piston approximations, nevertheless piston models have been commonly used historically [10], and often give indicative results [6]. A benefit of the piston model is that analytic expressions can be obtained for the complete sound field, along with the resulting directivity patterns of SPL [11].

3 Methods

We constructed two numerical simulations for two idealised drivers: (i) a circular piston in an infinite baffle, and (ii) a line array of seven pulsating spheres simulating a QRD phased array. Both of these idealised drivers admit analytic solutions for the anechoic steady-state sound field resultant from a monochromatic harmonic driver excitation. These solutions do not involve numerical integration terms, however the piston model does involve an infinite series expansion which must be computed numerically to a given precision. Nevertheless series expansion methods present some advantages in terms of efficiency and known convergence rates [11].

3.1 Piston Model

The sound field resulting from the harmonic excitement of a circular piston in an infinite baffle has been studied extensively (see [9] for a survey). A variety of numerical schemes — using either infinite series expansions or numerical integration — are available that describe the sound field entirely, either via analytic expressions for the velocity potential, velocity field, or pressure field. We examine the velocity field in detail; in particular the direction of particle velocity as a function of time and polar angle. For this purpose it is convenient to use Mast & Yu's [11] series expansion for the pressure field, and obtain an expression for the velocity field via the gradient of the pressure field according to the relation

$$\mathbf{v} = \frac{i}{k\rho_0 c}\nabla p$$

which holds for continuous wave harmonic excitation [6], where $k = \omega/c$ is the angular wavenumber, and ρ_0 is the ambient air density.

Then in the spherical polar coordinate system (r,θ,ϕ), and assuming without loss of generality that the azimuthal angle ϕ is zero (due to the cylindrical symmetry of the model), Mast & Yu give the following expression for the pressure:

$$p(r,\theta,t) \quad = \quad \sum_{n=0}^{\infty} \Lambda_n(ka) P_{2n}(\cos\theta) h_{2n}^{(1)}(kr)\,\mathrm{e}^{-i\omega t}$$

where the Λ_n are functions only of the ratio of the wavelength to the piston radius (see [11] for details).

Then, taking the gradient of the pressure field in spherical polar coordinates, and applying the above relationship between pressure field and particle velocity field and also utilising recursive identities for the derivatives of Legendre polynomials and spherical Hankel functions, velocity field expression becomes

$$\begin{aligned}\mathbf{v}(r,\theta) &= \frac{i}{k\rho_0 c}\sum_{n=0}^{\infty}\Lambda_n(ka)\\ &\times\Bigg[P_{2n}(\cos\theta)k\left(\frac{2n}{kr}h_{2n}^{(1)}(kr) - h_{2n+1}^{(1)}(kr)\right)\hat{\mathbf{r}}\\ &+\frac{\sin\theta}{r}\left(\cos\theta P_{2n}(\cos\theta) - P_{2n+1}(\cos\theta)\right)h_{2n}^{(1)}(kr)\hat{\theta}\Bigg]\end{aligned}$$

where $\hat{\mathbf{r}}$ and $\hat{\theta}$ and are radial and tangential unit vectors respectively.

3.2 QRD array model

The Quadratic Residue Diffuser (QRD) array was first applied to acoustics by Manfred Schroeder in 1979 [7]. He considered both phased arrays of spherical sources, and physical treatments of auditoria walls. Schroeder encouraged lateral diffusion as a source of binaural dissimilarity in auditorium design which he had found was a desirable property for a concert hall auditorium. In this approach the elements of the array are uniform in depth over the vertical height of the diffusers, a setup he described as a one dimensional (1D) array.

The QRD model presented in this paper mimics a 58mm wide one-dimensional diffuser based on the prime number 7. This was implemented by NewAudio Pty. Ltd. in the commercial loudspeaker system *AS8*, designed to create 7 spherical wave sources aligned at an 8.25mm inter-element width giving a total width of 58mm. We modeled a second QRD of 25mm element width and overall array 175mm width, and a third at 4mm element width for an overall array 29mm width. The corresponding time delays of the QRD were applied to the individual spherical sources in the modeled arrays.

The sound-field resulting from a line array of spherical drivers at various relative phases is relatively straightforward to calculate, simply being the (complex) sum of the individual spherical waves. Such arrays are discussed by, for example, Skudzrzyk [6], Pierce [12] and Schroeder [5]. Here we follow the method of Pierce.

Referring to figure (1), we calculate the polar patterns of sound pressure, sound intensity and particle velocity. Suppose that the field point of interest is positioned as shown at a polar coordinate of (r,θ) in the xz-plane. We assume that the surface of each sphere is oscillating with angular frequency ω, and write the velocity of

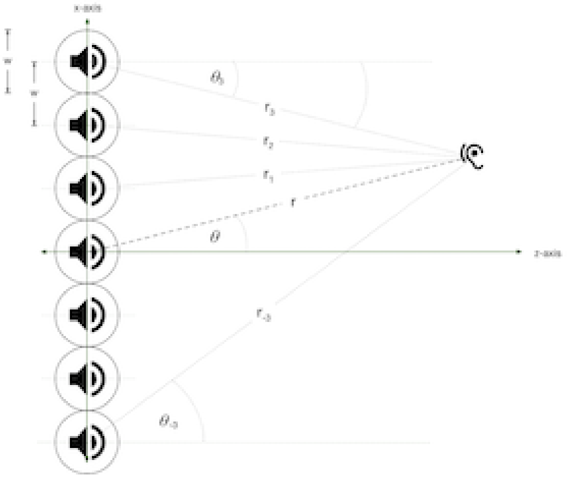

Fig. 1: Line array of seven drivers

the oscillation as $v = v_0 \cos(\omega t)$ where v_0 is the amplitude of the velocity oscillations. In this case, given the assumptions above, and for sufficiently high frequencies, the particle velocity $\mathbf{v}$, and the sound pressure p at position (r, θ) are

$$\mathbf{v} = \frac{v_0}{\rho c}\left(1 + i\frac{1}{kr}\right)\left(\frac{ika^2\rho c}{ika - 1}\right)\frac{e^{ik(r-a)}}{r}e^{-i(\omega t+\phi)}\left[\frac{\mathbf{r}}{r_n}\right]$$

$$p = v_0\left(\frac{ika^2\rho c}{ik - 1}\right)\frac{e^{ik(r-a)}}{r}e^{-i(\omega t+\phi)}$$

where c is the speed of sound, ρ the ambient air density, and $k = \frac{\omega}{c}$ is the angular wavenumber.

Referring again to figure (2), suppose that the individual drivers have phase delays of ϕ_n for $n = -3, ..., 3$. Writing $\mathbf{r}_n$ for the displacement vector from the centre of driver n to the listener, and $w = 2a$ for the width (and also the separation) of the drivers, then in terms of the original coordinate system

$$\mathbf{r}_n = \begin{bmatrix} r\sin(\theta) - nw, & 0, & r\cos(\theta) \end{bmatrix}^{\mathbf{T}}$$

and its magnitude r_n is

$$r_n = r\left(1 - \frac{2\sin(\theta)nw}{r} + \frac{n^2w^2}{r^2}\right)^{\frac{1}{2}}$$

For convenience we will define

$$\Phi(r, \theta, n) = \left(1 - \frac{2\sin(\theta)nw}{r} + \frac{n^2w^2}{r^2}\right)^{\frac{1}{2}}$$

so that $r_n = r\Phi(r, \theta, n)$. Then writing $R = \frac{r}{a}$ we obtain

$$\mathbf{v}_n = v_0\left(1 + \frac{i}{kaR\Phi}\right)\left(\frac{ika^2}{ika^2 - 1}\right) \times$$
$$\frac{e^{-ika^2R\Phi}}{aR\Phi}e^{-ika}e^{-i(\omega t+\phi_n)}\Phi^{-\frac{1}{2}}\begin{bmatrix} sin(\theta) - \frac{nw}{r} \\ 0 \\ cos(\theta) \end{bmatrix}$$

$$p_n = v_0\rho c\left(\frac{ika^2}{ika - 1}\right)\frac{e^{-ik\frac{w}{2}R\Phi}}{aR\Phi}e^{-i(\omega t+\phi_n)}$$

Then summing these individual driver contributions over $n = -3, \ldots, 3$ gives us the polar patterns for the pressure, and for the vector velocity.

4 Results

4.1 Circular Piston Tangential Motion

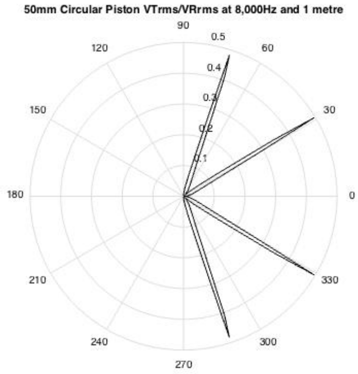

Fig. 2: VT/VR versus polar angle - circular piston model 50mm diameter - 8,000Hz.

The circular piston model of a 50mm diameter piston produced the polar plot of figure (2). The VT/VR ratio is around 48% to 50% at four polar angles (±31° and ±72°) off axis. Thus 4 large narrow lobes of tangential energy exist at 8,000Hz. At all other polar angles the tangential energy is relatively minimal.

As seen in figure (3), there is also a significant reduction in pressure at the polar angles where there is significant tangential velocity.

A potential metric of tangential energy acoustical distortion could consider the number of lobes and possibly

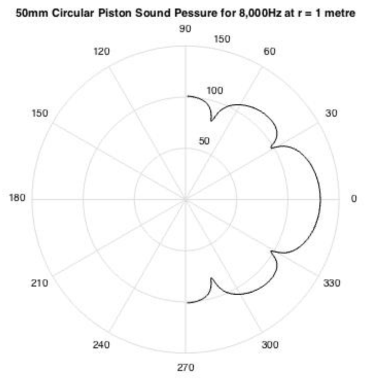

Fig. 3: Polar SPL - circular piston model 50mm diameter - 8,000Hz.

the area under these lobes, as well as the tangential sound intensity level, which will be lessened due to the polar alignment of these aberrant velocity spikes with dips in SPL directivity.

The 31° off axis particle velocity (PV) quiver of figure (4) shows the actual particle velocity direction as 57°. The nominal radial direction is represented by a solid line. The arctan of the VT/VR at 31° off axis lobe (50%) is 26°. Added to the nominal direction of 31° the result is 57° being the same direction of the actual particle velocity quivers. The PV quivers at 0° is purely radial (Quadrant top left). At 17° the PV quivers are showing signs of moving off the radial-only motion (Quadrant top right). At 31° the PV quivers are aligned a further 26° off the radial direction and exhibit an orbital pattern (Quadrant bottom left). At 72° the PV quivers are aligned beyond 90° and exhibit the same orbital pattern as the 31° PV quivers (Quadrant bottom right). It is these orbital shaped PV quivers that the authors suggest are a major source of acoustical spatial distortion in a sound reproduction system.

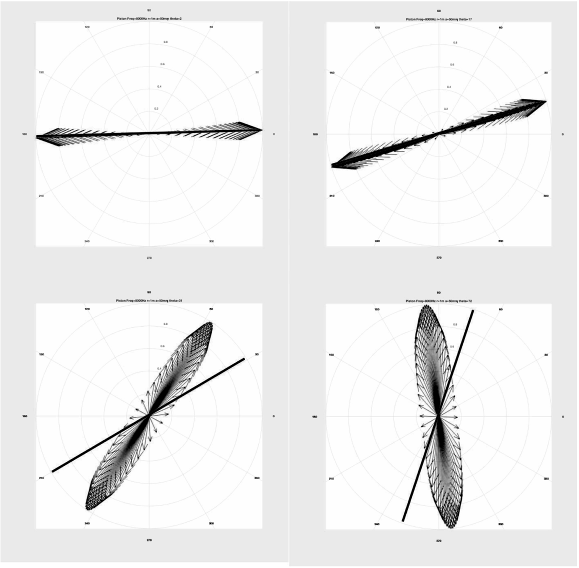

Fig. 4: Normalised Particle Velocity Quiver- circular piston model 50mm diameter - 8,000Hz/0°, 17°, 31°, and 72° off axis.

The tangential energy spike of figure (2) (50% at ±31° off axis) is shown again in figure (5) on the plot of the 50mm diameter circular piston model curve. At 8,000Hz the y-axis value is 50%. The curve for the 50mm driver shows a relatively low tangential energy at 4,000Hz and below. Above 5,000Hz the tangential energy is above 40% averaging 50%. The PV quivers, by definition, align at an angle given by the addition of the nominal radial direction plus the arctan(VT/VR), so that only low values of VT/VR produce PV quivers aligned substantially in the radial direction.

Similar curves exists for both the 150mm circular piston driver model (a typical mid/bass driver) and the 25mm circular piston model (a typical tweeter). The mid/bass shows significantly increased tangential energy at 1,600Hz and above, averaging 60%. The tweeter shows significant tangential energy at 10,000Hz and above, averaging 40%.

As the frequency increases, once the VT/VR% exceeds 30% it appears to stay high, at or above that reading. This means that, at all frequencies from this point on, unintended virtual sources will appear in the listening space at every frequency.

All three circular piston models (25mm, 50mm, and 150mm) display a critical frequency at which the tangential energy jumps from minor (« 10%) to major (» 10%), implying significant orbital PV quivers in the listening space above this critical frequency. If a radial motion is defined as say a tolerance within ±1° of the nominal radial direction then corresponding maximum value for VT/VR would be given by the tan(1°) = 1.7%.

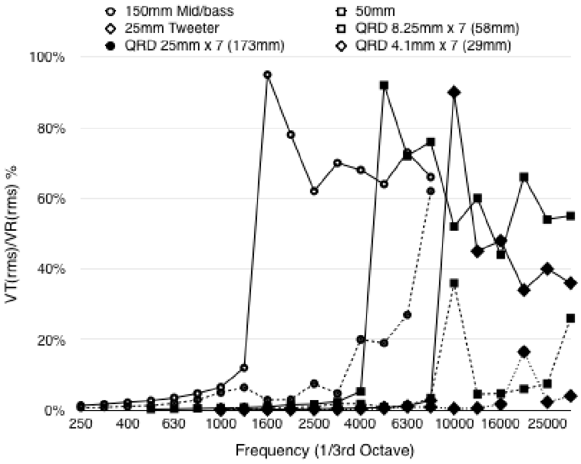

Fig. 5: Maximum VT/VR - piston versus QRD.

4.2 QRD Tangential Motion

Figure (5) shows the tangential behavior for three QRD models (29mm, 58mm, and 175mm) in comparison to the piston model at three similar widths. Immediately obvious are far lower VT/VR scores. At 10,000Hz, on the 58mm curve, the tangential component increases significantly. This is at what QRD designers call the 'plate' frequency — where the QRD behaves like a plate surface and ceases to diffuse. At this frequency the VT/VR ratio is 35%. The VT/VR% ratios are below 2% across most of the spectrum below this plate frequency. The 58mm wide array shows a VT/VR ratio of only 8% even at 20,000Hz. As stated there is an anomaly at 10,000Hz - the plate frequency. A second plate frequency exists above 20,000Hz.

The 29mm array shows a VT/VR% ratio below 2% up to 16,000Hz. Both plate frequencies exist above 16,000Hz. It is feasible to scale a QRD so as to avoid plate frequency within the audible spectrum. It is also feasible to use twin QRD arrays to overlap plate frequency regions, using crossover techniques to minimise tangential energy across the spectrum.

The 175mm QRD, of similar dimensions to the 150mm circular piston driver, shows a VT/VR ratio less than 8% up to 3,150Hz. The 150mm circular piston model has VT/VR of 70% at the same frequency (figure 5).

Figure (6) show PV quivers for a 58mm QRD design as used in the AS8. The particle velocity motion is substantially more radially aligned.

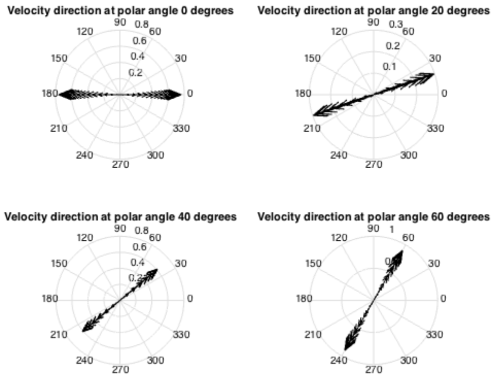

Fig. 6: Particle Velocity Quiver- QRD model 58mm diameter - 8,242Hz/0°, 20°, 40°, and 60° degrees off axis.

4.3 SPL Performance

The polar plots for piston radiators are well documented, and typically show a null or minima in the SPL lobe at differing angles depending on the frequency.

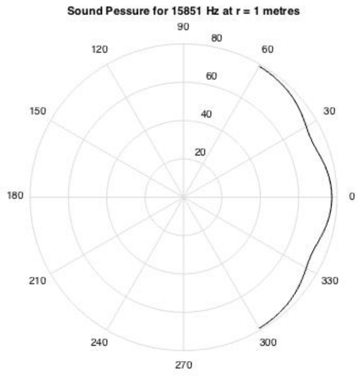

Fig. 7: Polar SPL - 29mm QRD model at 16,000Hz.

The polar SPL performance of the QRD array at 29mm and 16,000Hz is shown in figure (7). The SPL varies from 73dB to 79dB across a 120 degree polar angle. The maximum magnitude of the VT/VR ratio is only 1.7%.

As previously postulated, the dearth of tangential energy and good omni-directional polar SPL response are linked. The performance modeled in figure 7 would be quite acceptable as polar output for 16,000Hz from a high frequency driver. In this instance a 1.7% VT/VR ratio corresponds to a +/- 3dB SPL over a 120 degree polar wave front. A lateral diffusion, causing a lateral sound intensity = 0, we postulate is the equivalent of ‘silence’ with respect to inter-aural acoustic activity (ITD & IDD = 0). Consequently the only inter-aural activity will be derived from the multi-channel signal (e.g - stereo).

As the modeled radius to the microphone/listening position increases for the QRD, the overall VT/VR% reduces. This suggest that the QRD driven alignment, in the radial direction only, of particle velocities, improves with distance.

4.4 Validity of the Models

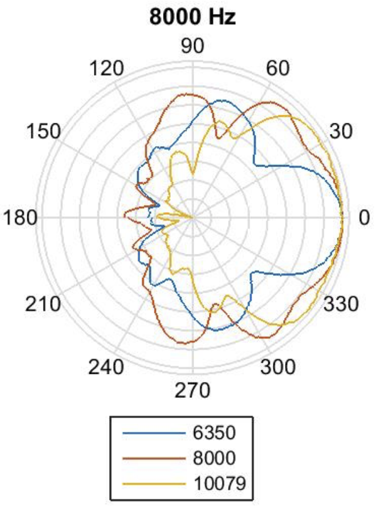

Fig. 8: Measured anechoic polar response SPL QRD 58mm - 6,350Hz, 8,000Hz, and 10079Hz.

For the piston model SPL polar patterns are well understood, and display significant lobes dependent on frequency and piston size, which correspond well to empirical measurements of many loudspeakers [6].

Figure (8) shows empirical measurements of the SPL polar patterns for an AS8 speaker, which uses the same QRD sequence as used in our model. The 8,000Hz plot in figure (8) between ±60° for a 58mm QRD correlates closely to the modeled plot, figure (7) being 16,000Hz at a 29mm (1/2 scale) QRD.

At present we do not have any empirical measurements of the particle velocity direction for either an A3D speaker or a conventional loudspeaker, as we do not have access to a suitable velocity probe. Comensaña et al. [13] investigated near-field particle velocities of an audio loudspeaker using a 3-axis velocity probe. They found substantial amounts of non-radial energy flow. Their study was to investigate noise contributions from speaker cabinet vibration; however they suggest that observed particle velocities arise from a combination of cabinet vibration and diffraction effects, but were unable to determine the relative contributions of these two phenomena. Their results, however, appear to be compatible with our theoretical findings.

5 Discussion

Loudspeaker-induced unintended sound sources, as discussed in this paper, add coloration and spatial cues that are likely at odds with the intention of the recording — being products purely of the speaker geometry. Ideally, there should be no unintended sound sources in the listening space. We suggest that near-zero tangential particle velocities are desirable in this respect.

Whilst creating tangential particle velocity is not the design intention of a loudspeaker, our models suggests that piston-like drivers create significant acoustical distortion in the form of this tangential energy. It may be best to avoid using these drivers above the critical frequency where their VT/VR percentage exceeds say 2% (an arbitrary indicator).

One possible advantage of minimal tangential acoustic energy in in-car loudspeakers, is that one might better predict their coverage. Delivering radial acoustic flow means that the SPL polar performance will be close to omni-directional. Within a confined space, such as a car interior, this may allow fewer speaker-drivers to be used with a more predictable sound field. The speaker-driver count and location count might be so reduced, perhaps even to a simple 2.1 arrangement.

‘Diffusion at source’ may also help mitigate specular reflections in the car interior. The QRD array appears to apodize (align radially) the acoustical particle velocities. The benefit of ‘diffusion at source’ appears to lie in its point spread function (PSF). It is close to an ideal radial-only particle velocity source, and hence its reflections should appear non-specular to the extent that they may have little deleterious effect on the low-frequency acoustic envelope, as is found for diffused reflections from treated walls [4].

6 Conflict of Interest Statement

Note that all of the authors have pecuniary interest in A3D. We seek to mitigate any potential bias by providing all the model details (in the sections above) and the source code used to generate these results (git@github.com:dr-offig/AA2017.git). Whilst we envisage commercial application of the A3D technology, we feel that our findings point to significant scope for additional useful research.

7 Summary

The sound field of a circular piston in an infinite baffle has been found to contain significant tangential energy in the polar pattern of particle velocity direction, particularly above a specific cone-dimension related critical frequency at which these ratios typically reach 50%. At higher frequencies these narrow lobes of tangential energy may appear as ‘unintended sound sources’ within the listening space. At high values of VT/VR they tend to exhibit an orbital rather than linear particle velocity motion.

The polar positions of large tangential energy (particle velocity motion) appear to be related to the ‘dips’ in SPL directivity patterns.

Contrasting these results with the sound-field of a linear array of spherical drivers with phase delays determined by a QRD sequence, we find that the QRD array yields greater uniformity in both SPL directivity and particle velocity direction, over a far larger portion of the audio spectrum.

Through careful design it is possible to minimise the ratio of VT/VR of a loudspeaker source such that it acts like an idealised monopole acoustic source wherein the only tangential acoustic energy created within the listening space is that created intentionally by stereo or other multi-channel audio signal. Such a low unintended-sound-source presence may be beneficial to in-car infotainment systems.

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